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Laboratory of combinatorial and geometric structures https://combgeo.org/eng,

Moscow Institute of Physics and Technology

Moscow, Russia

Margarita Akhmejanova

Education

Moscow Institute of Physics and Technology
PhD in Mathematics — 24 December 2021
supervisors: Prof. Dmitry Shabanov

Lomonosov Moscow State University (MSU)
Speciality (MSc+BSc): Mechanics and Mathematics,
Department of Probability Theory — September 2012 - June 2018

Research interests

Random graphs and hypergraphs, hypergraph colorings (proper, on-line, list etc.), concentration inequalities, weak saturation, probabilistic methods in combinatorics, extremal combinatorics, Ramsey theory.

Papers

[1] M. Akhmejanova, M. Zhukovskii, "EMSO (FO^2) 0-1 law fails for all dense random graphs", accepted to: *SIAM Journal on Discrete Mathematics*, preprint

[2] M. Akhmejanova, D. Shabanov, "Coloring hypergraphs with bounded cardinalities of edge intersections", *Discrete Mathematics*, 343, 2020, 111692. https://doi.org/10.1016/j.disc.2019.111692

[3] M. B. Akhmejanova, D. Shabanov, "Equitable colorings of hypergraphs with r colors", *Fundamental and Applied Mathematics*, 3:21, 2020, 323. http://mech.math.msu.su/~fpm/rus/k20/k201/k20101g.htm

[4] M. Akhmejanova, D. Shabanov, "Equitable colorings of hypergraphs with few edges", *Discrete Applied Mathematics*, 276, 2020, 212. https://doi.org/10.1016/j.dam.2019.03.024

[5] M. Akhmejanova, "On Equitable Colorings of Hypergraphs", *Mathematical Notes*, 106, 2019, 319326. https://doi.org/10.1134/S0001434619090013

[6] M. Akhmejanova, D. Shabanov, "Colorings of b-simple hypergraphs", *Electron. Notes Discrete*

Math, 61, 2017, 2935. https://doi.org/10.1016/j.endm.2017.06.017

[7] M. Akhmejanova, "Bi-uniform hypergraph", *Trudy MIPT*, 13:51, 2021 https://doi.org/10.53815/20726759_2021_13_3_23

Submitted papers

[8] M. Akhmejanova, J. Balogh, "Chain method for panchromatic colorings of hypergraphs", preprint

[9] M. Akhmejanova, I. Bogdanov, G. Chelnokov, "The continualization approach to on-line hypergraph coloring", preprint

[10] M. Akhmejanova, K. Olmezov, A. Volostnov, I. Vorobyev, K. Vorob'ev, Y. Yarovikov, "Wiener index and graphs, almost half of whose vertices satisfy Šoltés property", preprint

Awards

- Winner of the prestigious competition "Young Mathematics of Russia", **link** 2021
- laureate of the Moscow government scholarship 2021
- Diploma with honors, Mechanics and Mathematics, Moscow State University. 2018
- Moscow State University Excellence Scholarship for undergraduate students 2015-2018

Grants (co-principal investigator)

- Grant number MK-1451.2022.1.1 of the President of Russia 2022-2024 ~18 000US
- Grant number 22-21-00202 for small scientific groups 2022-2024 ~39 000US
- Grant number NSh-6760.2018.1 to Leading scientific schools of Russia 2018-2020 ~39 000US
- Grant number 22-21-00411 of the Russian Science Foundation 2020-2022 ~78 000US
- Grant number MD-1562.2020.1 of the Russian Science Foundation 2020-2022 ~25 000US

Research visits

IMPA Institute, *December 2021, Rio-de-Janeiro, Brazil*
visiting Prof. Robert Morris and Taísa Martins

Alfréd Rényi Institute of Mathematics *August 2021, Budapest, Hungary*
Visiting Prof. András Gyárfás and János Pach

Sobolev Institute of Mathematics *February 2019, Novosibirsk. Russia*
Visiting Prof. V. Potapov and Ann Taranenko

Selected talks

1. The 23rd Thailand-Japan Conference on Discrete and Computational Geometry, Graphs, and Games, (2021)

2. RSA, 15-19 July, (Zurich, Switzerland, 2019).
3. RSA, 7-11 August, (Gniezno, Poland, 2017).
4. EUROCOMB 28 August--1 September, (Vienna, Austria, 2017).
5. 2nd Russian-Hungarian Workshop, June 27--29, 2018, (Budapest, Hungary, 2018).
6. Lomonosov-2017, (Russia, Moscow, 2017).
7. 1st Russian-Hungarian Workshop on Discrete Mathematics, (Russia, Moscow, 2017)
8. Lomonosov-2018, (Russia, Moscow, 2018).
9. 61st All-Russian Scientific Conference of MIPT, (Russia, Dolgoprudny, 2018).
10. Extreme combinatorics and discrete geometry, (Russia, Maikop, 2018).

Teaching and work experience

Laboratory of combinatorial and geometric structures junior research, https://combgeo.org/en/	January 2020-present
laboratory for advanced combinatorics and network applications, MIPT, engineer	January 2018-present
Department of Discrete Mathematics, MIPT, Teacher	Autumn 2019
MSU, Assistant	Autumn 2019

List of references

These people have kindly agreed to give me recommendations and have already prepared a letter of recommendation.

Maksim Zhukovskii zhukmax@gmail.com
Jozsef Balogh jobal@illinois.edu

Also, your colleague, Prof. Jakub Kozik, knows me a little.

Research impact:

1. Coloring hypergraphs with bounded cardinalities of edge intersections

In paper https://doi.org/10.1016/j.disc.2019.111692 we consider the family of b-simple hypergraphs, in which any two edges do not share more than b common vertices. We proved that for $n \geqslant n_0(b)$, any n-uniform b-simple hypergraph with the maximum edge degree at most $c \cdot nr^{n-b}$ is r-colorable. The work was approved by Jacub Kozik and Danila Cherkashin.

2. Equitable and panchromatic hypergraph colorings.

A vertex coloring of a hypergraph is called panchromatic if every edge sees every color. The main result of paper preprint is a new lower bound on the quantity $p(n, r)$, which is the minimum possible number of edges in an n-uniform hypergraph that does not admit a panchromatic coloring with r colors. This is a well-studied parameter, which was first considered by Kostochka in 2002; he established lower and upper bounds for the function. The result significantly improves on the lower bound known before.
In paper https://doi.org/10.1016/j.disc.2019.111692 we give a new lower bound on $m^*(n, r)$, which is the minimum possible number of edges in an n-uniform hypergraph that does not admit an equitable coloring (proper + equality of color classes in total) with r colors. The proof relies on a Pluhar's technique, but the significant part of the proof is devoted to the building equitable coloring from proper coloring. The bound for $r < \ln n$, coincides up to a constant with the same bound for proper r-colorings.

3. *A continulization approach to the on-line hypergraph coloring*

2-colorability of hypergraphs, also known as “property B”, admits on-line version. In on-line version vertices get its colors on-line, i.e. on step i, Painter gets the information about the subset of edges which contain vertex v_i and must immediately assign color black or white to the presented vertex. On-line coloring can be solved by building a bijection between on-line coloring and chip game, first suggested by Spenser. We introduced a general version of chip game and its continuous version. Unlike discrete chip game, it turns out that continuous one gives precise answers. We got in preprint precise results for continuous version of proper and panchromatic r-colorings, and also for list colorings of bipartite graphs.

4. EMSO (FO^2) 0-1 law fails for all dense random graphs

In paper preprint, we disprove $\text{EMSO}(\text{FO}^2)$ convergence law for the binomial random graph $G(n,p)$ for any constant probability p. More specifically, we prove that there exists an existential monadic second order sentence with 2 first order variables such that, for every $p \in (0,1)$, the probability that it is true on $G(n,p)$ does not converge

Current scientific projects

1. 2-point concentration of some structures in G(n,p) (Zhukovskii, Kozhevnikov)

It is known that for some p independence number, largest size of the induced subgraph with a given number of edges, the largest size of the induced subgraph with a given restriction on the number of edges, the largest size of induced matching etc are concentrated in two consecutive points whp. Now we are managed to prove 2-point concentration for the largest size of an induced tree with a bounded degree and working on generalizing these results to sparse settings.

2. Weak saturation number of bipartite graphs (Zhukovskii et al.)

Let G and F be graphs. A spanning subgraph $H \subset G$ is weakly F-saturated in G if it contains no copies of F and there exists an ordering $e_1, \dots, e_t$ of all edges of $E(G) \setminus E(H)$ such that the addition of e_i to $H \cup \{e_1, \dots, e_{i-1}\}$ creates a new copy of F for all $i \in [t]$. The minimum number of edges in a weakly F-saturated subgraph of G is denoted by wsat (G, F). In 1968 Bollobás conjectured that wsat $(K_n, K_s) = \binom{n}{2} - \binom{n-s+2}{2}$. The conjectured was confirmed by Lovász. In 1985 Kalai proved that wsat $(K_n, K_{s,s}) = (s-1)(n+1-s/2)$ for all $n \geq 3s-3$. This result has been reproven by Kronenberg, Martins and Morrison. Now we work on the case $2s < n < 3s-3$.

3. Continuous version of chip game and its application to on-line hypergraph coloring (Bogdanov, Chelnokov)

It is my favorite project. The paper deals with an algorithmic problem concerning combinatorial game theory. Here we introduce and analyze a continuous generalization of Chip Game. The general Chip game was introduced by Aslam and Dhagat to model on-line type problems on hypergraph coloring.

On-line property B: given a hypergraph $H = (V, E)$. Painter does not know the hypergraph H, but he knows the set of edge cardinalities, i.e. multi-set $\mathcal{A}(H) = \{|e| : e \in E(H)\}$. Let vertex set V be enumerated by $\mathbb{N}$. In round i, Painter gets the information about the subset of edges which contain vertex v_i. Painter must immediately assign color black or white to the presented vertex v_i. Painter wins when all vertices have been colored and no edge is monochromatic. Denote this game by $(\mathcal{A}, 2)_{ol}$ game. For which multisets of edge cardinalities $\mathcal{A}$ Painter has a winning strategy in $(\mathcal{A}, 2)_{ol}$ game?

Here we introduce and analyze a continuous generalization:

General Gold Sand game $(S, \tau_{i\in\mathcal{F}})$ is determined by a finite set S, called the set of paths, an element dead $\in S$, called death path, and a set of mappings $\tau_{i\in\mathcal{F}}: S \to S$, satisfying $\tau_i(\text{ dead }) = \text{dead}$ (once dead stays dead). The cells of the playing field are enumerated by $(\mathbb{N} \cup \{0\}) \times S$, the cells $(0, \text{path } m) : m \neq$ dead are called winning cells. All cells contain real non-negative numbers, called gold sand. There is only finite amount of non-zero numbers. In each round Pusher splits gold sand in each cell into two parts, standing and running. Then Remover (knowing how Pusher shared and knowing the amount of gold sand in each cell) picks one of the mappings τ_i. After that each standing part keeps its cell, and each running part changes its cell according to the rule $(n, \text{path } m) \to (n-1, \text{path } \tau_i(m))$. Moreover, all sand from the winning cells instantly becomes Pusher's win and is removed from the field. Then the new round begins.

The question: let $X = \left(x_{i,path\ j}\right)_{i,j}$ denotes the matrix of initial distribution of gold sand, in short, arrangement X. Let $\mathcal{X}$ be the set of all matrices of initial distribution of gold sand. Can one find the supremum of Pusher's win for any given arrangement $X \in \mathcal{X}$?
Our result: Consider the function $g: \mathcal{X} \times [0,1]^r \mapsto R_+$, defined by $g(X,p) = \sum_{j=1}^{N} \left(p^j x_{j,path} 1 + (1-p)^j x_{j,path} 2 + (p^j + (1-p)^j) x_{j,path0}\right)$. Then the supremum of Pusher's win in on-line 2-coloring game is the minimum of $g(X,p)$ on $0 \leq p \leq 1$. We also have precise results for proper r-coloring, panchromatic r-colorings and list colorings of K_m,m.

4. *Coloring of non-uniform hypergraphs (Martins, was suggested by Kozik)*

For a hypergraph H, let $g(H) = \sum_{e \in H} 2^{-|e|+1}$ denote the expected number of monochromatic edges when the color black or white of each vertex is sampled at random. Let $s_{\min}(H)$ denote the minimum size of an edge in H. Erdos asked in 1963 whether there exists an unbounded function $g(n)$ such that any hypergraph H with $s_{\min}(H) \geqslant n$ and $q(H) \leqslant g(n)$ is two colorable. Beck in 1978 proved that $g(k) = \Theta(\log^* n)$. In 2018, Kozik, Duraj and Gutovskii proved that $q(H) = \mathcal{O}(\log n)$. We study the local variant (edge degree) of this problem by considering the functions

$$g(H) = \max_{c \in H} \sum_{j \in H:/nc \neq \emptyset} 2^{-|J|+1}$$

$$g(n) = \min\{g(H): H \text{ with } S_{\min}(H) \geq n \text{ and } \chi(H) > 2\}$$

We think, we proved that ``if H a simple non-uniform hypergraph with minimum size of an edge is at least n and $g(H) < n$, then H is 2-colorable''.

5. *Turan type problem on maximum number of edges in linear 3-uniform hypergraph without path with k edges (Kozhevnikov, was suggested by Gyárfás).*

A linear Turán number $\text{ex}_L(n, F)$ of a linear 3-hypergraph F is defined as the maximum number of edges in a linear 3-hypergraph on n vertices without copies of (subgraphs isomorphic to) F. Gyárfás et al. initiated the study of linear Turán number of acyclic graphs, particularly, linear paths. Known facts (Gyárfás et al.):

$$\text{ex}_L(n, T_k) \leq (2k-3)n, \qquad \text{ex}_L(n, P_k) \leq 1.5\text{kn}.$$

Gyárfás et al. believe that the bound $\text{ex}_L(n, P_k) \leq 1.5kn$ is far from tight. We think, we can slightly improve this bound

$$\text{ex}_L(n, P_k) \leq 1.4\text{kn}$$

6. *The size if shadow in 3-wise intersecting families (Anika Kuchukova, was suggested by G. Katona)*

When I was in Hungary, G. Katona kindly suggested to me some exercises. E.g. if family of k-sets is intersecting, then the ratio between the size of shadow and the size of family is at least 1. Can one show that if family of k-sets is 3-wise intersecting (no 3 pairwise disjoint sets) then the ratio is at least 1/2?

7. *Sum of all distances between all pairs of vertices in G(n,p) has normal distribution (Vorobyev)*

In chemistry, the Wiener index $\mathbf{W}(\mathbf{G})$ is defined as the sum of the lengths of the shortest paths between all pairs of vertices in the chemical graph **G** representing the non-hydrogen atoms in the molecule. It has been studied in many graphs. E.g, Egorov and Vesnin investigated the correlation of hyperbolic volumes of fullerenes with the Wiener index. Using Bollobas results we found good bounds for $\mathbf{W}(\mathbf{G}(\mathbf{n},\mathbf{p}))$ and now work on a much more difficult task, namely, on proving that it has normal distribution.

8. *Fractional hypergraph coloring*

Given a graph, integers $\mathbf{0} < \boldsymbol{b} \leq \boldsymbol{a}$, and a set of $\boldsymbol{a}$ colors, a proper $\boldsymbol{a}/\boldsymbol{b}$-coloring is a function that assigns to each vertex a set of $\boldsymbol{b}$ distinct colors, in such a way that adjacent vertices are assigned disjoint sets. The fractional chromatic number of a graph, $\boldsymbol{\chi_F}(\boldsymbol{G})$, is the infimum of all rational numbers $\boldsymbol{a}/\boldsymbol{b}$ such that there exists a proper $\boldsymbol{a}/\boldsymbol{b}$-coloring of $\boldsymbol{G}$. Here we study famous Erdős–Hajnal problem of finding $\mathbf{m}(\mathbf{n})$ number for fractional hypergraph colorings.