Steiner’s Porism Open Problem:

Descartes's theorem (1643) states that in the plane the curvatures of four mutually touching circles satisfy a certain quadratic equation. The analogue of this theorem for n + 2 spheres in n dimensions is known as the Soddy-Gosset theorem. Generalize Descartes's theorem for the cases when touching spheres also touch two non-overlapping spheres and the contact graph of the spheres is

i) cross-polytope (for all n),
ii) icosahedron (n = 3),
iii) 600-cell (n = 4).

For more information see O. R. Musin, Analogs of Steiner's porism and Soddy's hexlet in higher dimensions via spherical codes, Arch. Math., 111 (2018), 493501.